Demand Creation and Economic Growth

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ABSTRACT

In the standard literature, the fundamental factor to restrain economic growth is diminishing returns to capital. This paper presents a model in which the factor to restrain growth is saturation of demand. We begin with common observation that growth of an individual product or sector grows fast at first, but its growth eventually declines to zero. The economy sustains growth by the introduction of new products/industries. Preferences are endogenous in this model. The introduction of new products/industries affects preferences, and creates demand. By so doing, it induces capital accumulation, and ultimately sustains economic growth.

JEL Nos. E1, E2

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In the standard literature, the fundamental factor to restrain economic growth is diminishing returns to capital. In this paper, by presenting a model we suggest that 'saturation of demand' is another important factor to restrain growth.

In the less mathematical literature and casual discussions, the idea of 'demand saturation' has been very popular. In fact, plot a time series of production of any representative product such as steel and automobile, or production in any industry against year, and with few exceptions, one obtains a S-shaped curve. Figure 1 due to Rostow [1978] demonstrates this 'stylized fact'. The experiences of diffusion of consumer durables such as refrigerator, television set, car, and personal computer tell us that deceleration of growth comes mainly from saturation of demand rather than diminishing returns in technology. Growth of production of a commodity or in an individual industry is bound to slow down because demand grows fast at the early stage but eventually its growth necessarily slows down. Thus the demand for some products grows much more rapidly than the GDP, while the demand for others grows much more slowly. Products/industries face different income elasticities of demand. The celebrated Engel Law based on saturation of demand for food is merely an example.

Unfortunately, the existing literature on growth abstracts largely from this important fact that products/industries face different income elasticities of demand and that each product/industry experiences a typical S-shaped life cycle; The standard assumption in the literature is that all the products are 'symmetric' and income elasticities of demand are common for all the products, namely that the elasticities are one for all. This standard assumption simply contradicts common observations such as the Engel Law and Figure 1. In place of it, this paper takes the logistic growth of an individual product/industry as a 'stylized fact', and presents a formal model of growth built on this 'stylized fact'.

An obvious implication of the logistic growth of an individual product/industry is that the economy enjoys high growth if it successfully keeps introducing new products or industries for which demand grows fast. Despite its popularity, however, the idea of demand saturation/creation has never been formally analyzed presumably because of apparent mathematical difficulty. In this paper, we postulate that growth of each commodity or sector decelerates over time and goes asymptotically down to zero. The 'age' of a product or sector is essential in our analysis. Innovations based on learning by doing in production bear new commodities or sectors which enjoy high growth of demand, and by so doing sustain economic growth of the economy as a whole.

We do not mean to suggest that our model is a substitute for the existing literature, but believe that it sheds lights on a neglected and yet very important aspect of innovations and economic growth. The existing literature focuses on the issue of whether the elasticity of capital in production function is one (the so-called AK model) or less than one (the 'old' Solow model). It concerns the property of production function or technology. We maintain that not only diminishing returns on capital in production but also saturation of demand for existing products/sectors is a very important factor to limit economic growth. We also maintain that in addition to the standard total factor productivity (TFP) growth, namely an 'upward shift' of production function, technical progress creates demand.

To substantiate this argument, in section I, we present a model which incorporates the basic idea. We begin with demand for individual product rather than preferences since the former is more directly related to the stylized fact than the latter. Section II studies growth of the economy as a whole. Out of steady state, 'vigor of demand' determines growth while the ultimate factor to sustain economic growth in steady state is creation of new products/industries. Under the standard Poisson assumption, successive creation of new products/industries sustains steady state growth. However, we demonstrate that under the alternative 'Polya urn' assumption that the success probability of innovation gets smaller as time goes by, the growth rate of the economy must decelerate and go asymptotically down to zero. Section III provides microeconomic foundations for demand. By so doing it suggests two different macroeconomic models, one the Ramsey model with the representative consumer, and the other with diffusion of goods among different households. Two models suggest different interpretations of saturation of demand. Finally, section IV offers some concluding remarks.

I. The Model

We study an economy in which heterogeneous final goods and an intermediate good are produced. In this section, we take demand for each final product as given, and concentrate on production. Section III considers the consumer's behavior and provides microeconomic foundations for demand. Let us begin with final goods.

A. Final Goods

Final goods are produced with an intermediate good as the only input. Production of all the final goods requires the same intermediate good X. Production function is also common.

(1) $y_k = AX_k \qquad (0 < A < 1)$

We assume the perfect competition. Therefore, zero profits ensue:

(2) $P_k(t)y_k(t) = P_X(t)X_k(t)$

Here $P_k(t)$ is the price of the k-th final product, and $P_X(t)$ the price of intermediate good.
Because of the common linear production function (1), the zero profit condition (2) is equivalent to

(3) $P_k(t)A = P_X(t)$

Thus, we can adjust unit of final products in such a way to make all the prices of final goods one. Then

$$P_X = A < 1$$

Output of each final good is equal to its demand $D_k(t)$ no matter how the latter is determined.

(4) $y_k(t) = D_k(t)$

In this section, we take a S-shaped life cycle of demand for each product/industry as a stylized fact. To make our analysis tractable, assume that $D_k(t)$ follows the logistic curve:

(5) $$D(t) = \frac{mD_0}{\left[dD_0 + (m - dD_0)e^{-mt}\right]}$$

Since the mechanism is the same for all the products or sectors, for the moment, we drop k for clearness and write $D_k(t)$ as $D(t)$. We will explore microeconomic foundations for the logistic growth of demand in section III; In section I and II, (5) is taken as given. D_0 in (5) is the initial value of $D(t)$. Starting with D_0 smaller than m/d, $D(t)$ initially increases almost exponentially, but its growth eventually decelerates, and approaching its 'ceiling' m/d, the growth rate declines asymptotically to zero. A typical shape of the logistic growth is illustrated and compared with exponential growth in Figure 2.

There are several ways to interpret the logistic growth. One way is to interpret it as the expected value of a particular stochastic process. Consider, for example, the following birth and death process. With an appropriate unit u (u>0), we can write $D(t) = un_t$ for integer n_t. $D(t)$ instantaneously changes by either u or $-u$. And assume that the probability that $D(t)$ increases from nu to $(n+1)u$ between t and $t + \Delta t$ is $mn\Delta t + o(\Delta t)$ while the probability that $D(t)$ decreases from nu to $(n-1)u$ is $dn^2\Delta t + o(\Delta t)$ with both μ and d positive. This is a typical birth and death process. Note that the 'birth rate' μ is constant while the 'death rate' dn is increasing in n. The idea is that when a product or an industry gets older and n becomes larger, the probability that the product is replaced by other new products or the industry falls behind new industries becomes higher. We shortly show that an alternative specification is possible.

We write the probability that $D(t) = un$ as $P(n, t)$. Then under the above assumptions, $P(n,t)$ satisfies the following differential equation:

(6) $$\frac{dP(n,t)}{dt} = m(n-1)P(n-1,t) + d(n+1)^2 P(n+1,t) - (m + dn)nP(n,t)$$

We denote the expected value (the first moment) of $n(t)$ or equivalently that of $D(t)$ by $\hat{D}(t)$; i.e.

$$\hat{D}(t) = \sum_{n=1}^{\infty} nP(n,t)$$

Then given (6), $\hat{D}(t)$ satisfies the following differential equation[1]:

$$\frac{d\hat{D}(t)}{dt} = \sum_{n=1}^{\infty} n \frac{dP(n,t)}{dt} \tag{7}$$
$$= m\hat{D}(t) - d\hat{D}(t)^2$$

Equation (7) can be solved to obtain the logistic equation (5); $D(t)$ in (5) is to be interpreted as the expected value $\hat{D}(t)$ of $D(t)$ in this birth and death process.

To obtain the logistic equation for $\hat{D}(t)$, we assumed above that the 'birth rate' µ was constant while the 'death rate' dn_t was increasing in n_t. Alternatively we can assume that the birth rate m / n_t is decreasing in n_t while the death rate d is constant. Diffusion of a product may lead to eventual deceleration of growth of demand as is often the case for consumer durables, and/or room for quality improvement of a product which presumably raises demand for it may get narrower as the product becomes older. In any case, under this alternative assumption of m / n_t and d, $\hat{D}(t)$ satisfies the following equation:

$$\frac{d\hat{D}(t)}{dt} = m - d\hat{D}(t)$$

instead of (7). Since the qualitative results obtained are basically the same as for the logistic case, in what follows we will keep the logistic assumption. We simply note that the diminishing 'birth rate' of demand can be handled in a similar way.

Though exponential growth is often taken for granted by economists, there is actually ample evidence to show that no *individual* product or industry grows exponentially. Rather demand for or production of a product, or an industry typically grows drawing the logistic curve. In fact, an eminent mathematician Montroll [1978] goes so far as to suggest that almost all the social phenomena, except in their relatively brief abnormal times, obey the logistic growth. Figure 1 well demonstrates this well-known fact of life in our economy.

Production of each final product $y_k(t)$ follows the logistic growth of demand; $y_k(t)$ also satisfies equation (5). So far, we have focused on a final good. The number of final products is not given, however. Rather at every moment a new product or sector emerges. The emergence of an utterly new final good or a new sector is the result of innovations. Before we explain it, we turn to production of intermediate goods taking the number of final goods *N* as if it were constant.

B. Intermediate Good

To keep our model as simple as possible, we assume that there is only one kind of intermediate good *X*, and that *X* is produced by using capital *K* alone:

(8) $X = K.$

Here *X* is the sum of intermediate goods used in production of final goods:

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[1] To be precise, $\hat{D}(t)$ satisfies

$$\frac{d\hat{D}(t)}{dt} = \sum_{n=1}^{\infty} n \frac{dp(n,t)}{dt}$$
$$= m\hat{D}(t) - d\hat{D}(t)^2 - dV(t).$$

$V(t)$ is the variance of $n(t)$ or $D(t)$ with the differential equation defined in terms of $\hat{D}(t)$, $V(t)$ and $K_3(t)$ where $K_3(t)$ is the third central moment or kurtosis of $n(t)$ or $D(t)$. It can be shown that with $\hat{D}(\infty) = m / d$, both $V(\infty)$ and $K_3(\infty)$ are zero.

$$X = \sum_{k=1}^{N} X_k$$

We note that production function (8) has unitary elasticity of capital, and therefore that as long as capital accumulates, X grows without limit. And given the common function of final good (1), whenever X grows, production of final good can also grow. However, X is intermediate good, and as we have seen it previously, growth of demand for each final good decelerates and declines eventually to zero. In this model, the factor to limit growth is not diminishing returns on capital but declining growth of demand.

Capital accumulates so as to maximize the value of this industry (firm). Profit of this industry stems from selling intermediate goods to firms producing final goods, and is

(9) $$P_X X(t) = P_X K(t)$$

Investment, namely an increase in K on the other hand requires finished goods as an input. To be specific, we assume that an increase in K at the rate of z requires $j(z)$ of final products. $j(z)$ satisfies

(10) $$j'(z) > 0, \qquad j''(z) > 0 \quad \text{for } z = \dot{K}/K \geqq 0 \text{ with } j(0) = 0, \qquad j'(0) = 1.$$

For simplicity, we assume that capital does not depreciate. Assumption (10) means the standard convex adjustment cost of investment. Romer [1986], among others, makes this assumption.

The value of this industry (or firm) S is then given by

(11) $$S_t = \int_t^\infty \left[P_X K_t - j(z_t) K_t\right] \exp\left\{-\int_t^t r_u du\right\} dt$$

where $j(z)K$ is investment. We note that S_t satisfies

$$r_t = \frac{\dot{S}_t}{S_t} + \frac{(P_X - j(z_t))K_t}{S_t}$$

and observe, therefore, that r_t is the rate of return on 'stock' of this firm or the interest rate. Given the initial capital stock K_0, the firm maximizes (11) under the assumptions in (10). Since $j(z)$ is convex, the optimum capital accumulation z_t is uniquely determined when it exists (Uzawa [1969]). By means of the maximum principle, we know that it satisfies

(12) $$j'(z_t) = \int_t^\infty \left[P_X - j(z_t)\right] \exp\left\{-\int_t^t (r_u - z_u) du\right\} dt$$

The right-hand side of (12) can be interpreted as Tobin's marginal q; For optimality of the firm's investment decisions, the marginal cost of investment must be equal to the marginal q. With production function (8), it is plain that growth of intermediate goods also satisfies (12). Given (1), so does the growth of final goods.

C. Emergence of New Final Goods or Industries

So far we have taken the number of final goods as if it were constant. In fact, new final goods and/or industries always emerge as a result of innovations. We can flexibly interpret final 'goods' as 'sectors' or 'industries' if we wish.

Much effort has been made to explicitly analyze R and D activities and inventions in growth models. In fact, the achievement of the 'endogenous growth theory' is to have combined growth models with models of R & D activities. Certainly, in the advanced economies a substantial part of R and D is done by private firms from profit motives. It is not the whole story, however, because R and D of non-profit motives also plays an important role. Research activities at universities is an obvious example. In Japan (1996), for example, out of 15 trillion yen annual national R & D expenditures, 3 trillion yen finances researches at colleges and universities, 2 trillion yen goes to public research institutions, and the remaining 10 trillion yen is spent by private firms for their R & D activities. Roughly one third of the broadly defined R & D activities are done by non-profit motivated institutions. In many countries, government also spends considerable money on R & D. According to the OECD *Basic Science and Technology Statistics*, the share of public money in total R & D expenditures is 33.6% for U. S. (1995), 22.9% for Japan (1995), 37.1% for Germany (1991), and 44.3% for France (1993). We also note that borrowed technology plays an important role for late comers, and that borrowed technology is not necessarily fully patented.

In any case, profit-motivated R & D activities have been already well analyzed in the existing literature. Grossman and Helpman [1991], for example, present 'product variety' and 'quality ladder' models. In the former, profit-motivated R & D activities keep generating new products, and an expanding product variety, by way of the so-called Dixit/Stiglitz utility/production function, brings about ever higher level of utility or production. In the latter, given the number of commodities, R & D investment improves quality of the commodities. Aghion and Howitt [1992], on the other hand, present a model in which successful innovations raise economywide efficiency.

These works no doubt shed much light on important aspects of innovations and economic growth. However, given the complexity of the way in which technical progress affects economic growth, much remains to be done. The existing literature based on the Dixit/Stiglitz production/utility function or otherwise, in effect, endogenously explains total factor productivity. In what follows, we make an attempt to formalize the hitherto neglected aspect of technical progress, namely 'demand creation' due to technical progress, and study how it affects economic growth. Our primary interest is not in microeconomic foundations for R & D activities but in the way in which technical progress affects the economy. Not to minimize the importance of profit motives of private firms to do R and D but to simplify the analysis for our purpose to focus on a different problem, the present analysis following Arrow [1962] and Stokey [1988] abstracts from profit maximization.

We assume that an invention of a new final good or an emergence of a new sector stems stochastically from learning in the process of production of the existing products. To be specific, we assume that the probability that a new final good is invented or a new industry emerges between t and $t + \Delta t$ is $lN\Delta t$ where N is the number of existing final goods (?>0). Since an invention or an emergence of new sector is a branch off from an existing good or sector, the rate of success probability is proportional to the number of existing final goods/sector N; The more the number of products or sectors in the economy, the more likely a new product or sector emerges. ? is a parameter to represent the strength of innovations or more precisely the probability that a new good or industry emerges in the existing process of production. Innovations are thus accidental, but the prior 'knowledge' and experiences which stem from the existing production is essential to them. In this respect, we follow Arrow [1969] who argues that "the set of opportunities for innovation at any one moment are determined by what the physical laws of the world really are and how much has already been learned and is therefore accidental from the viewpoint of economics".

Given this assumption, the probability that the number of final goods at time t, $N(t)$ is equal to N, $Q(N, t)$ satisfies the following equation.

(13) $$\frac{dQ}{dt} = -lNQ(N,t) + l(N-1)Q(N-1,t)$$

The appendix shows that the solution of this equation under the initial condition

$$Q(N,0) = d(N - N_0) = d(N-1)$$

is

(14) $$Q(N,t) = e^{-lt}(1-e^{-lt})^{N-1}$$

The probability that there are N goods at time t and the N + 1-th good emerges during t and $t + \Delta t$ is then given by

(15) $$lNQ(N,t)\Delta t = lNe^{-lt}\left(1-e^{-lt}\right)^{N-1}\Delta t$$

At time t, the production of final good which emerged at $t\,(t < t)\, y_t(t)$ has grown to

(16) $$y_t(t) = \frac{m}{d + (m - d)e^{-m(t-t)}}$$

since the growth of $y_t(t)$ obeys the logistic curve. Without loss of generality we can assume that the initial production of newly invented good D_0 to be 1 in equation (5). This is the structure of the economy.

II. Growth of the Macroeconomy

In this section, we will analyze the growth of the macroeconomy in this model.

A. The Basic Result

The aggregate value added or Gross Domestic Product (GDP) of this economy is stochastic, but in what follows, we will focus on its expected value and denote it by $Y(t)$. $Y(t)$ is simply the sum of production of all the final goods. Since profits in the final good sectors are zero by the assumption of perfect competition, the aggregate value added is equal to the value added (profit) of the intermediate good sector, $P_X X(t)$ which is equal to $\sum_k AX_k = \sum_k y_k$.

Figure 3 illustrates this model economy. Each sector once it emerged grows logistically. New sectors emerge stochastically, and the aggregate value added or GDP is simply the sum of outputs of all the then existing sectors.

From (15) and (16), we know that the expected value of GDP of this economy is given by

(17)
$$Y(t) = \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} y_t(t)dt + \frac{m}{(d+(m-d)e^{-mt})}$$
$$= \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} \frac{m}{\left[d+(m-d)e^{-m(t-t)}\right]} dt + \frac{m}{(d+(m-d)e^{-mt})}$$

The second term of the right hand side is simply output of the 'first' sector at time t, $y_0(t)$. Using

$$lNe^{-lt}(1-e^{-lt})^{N-1} = \frac{d}{dt}(1-e^{-lt})^N$$

and

$$\sum_{N=1}^{\infty}(1-e^{-lt})^N = e^{lt} - 1$$

we obtain

$$Y(t) = \int_0^t \frac{\left[\frac{d}{dt}(e^{lt}-1)\right]m}{\left[d+(m-d)e^{-m(t-t)}\right]} dt + \frac{m}{(d+(m-d)e^{-mt})}$$

(18)
$$= l\int_0^t \frac{e^{lt}m}{\left[d+(m-d)e^{-m(t-t)}\right]} dt + \frac{m}{(d+(m-d)e^{-mt})}$$
$$= l\int_0^t \frac{e^{l(t-u)}m}{\left[d+(m-d)e^{-mu}\right]} du + \frac{m}{(d+(m-d)e^{-mt})}$$

From (18), the growth rate of GDP becomes

$$g_t = \frac{\dot{Y}(t)}{Y(t)} = l + \left(\frac{f(t)}{Y(t)}\right)\left(\frac{\dot{f}(t)}{f(t)}\right)$$

where $f(t)$ is the logistic equation:

$$f(t) = \frac{m}{(d+(m-d)e^{-mt})}$$

It is easy to show that g_t satisfies

(19) $$\dot{g}_t = (g_t - l)[2(m-d)e^{-mt}f(t) - m - g_t]$$

with initial value g_0.

$$g_0 = \frac{\dot{Y}(t)}{Y(t)}\Big|_{t=0} = l + m - d$$

Also, since $e^{-mt}f(t)$ approaches zero, we can establish that the growth rate of GDP asymptotically approaches ?.

$$\lim_{t\to\infty} g_t = \frac{\dot{Y}(t)}{Y(t)} = l$$

The growth rate of the economy is initially higher than ? by µ- d, but it eventually goes down to ?. The exact time path depends, of course, on µ, d, and ?.

It is important to recognize that not only the steady state growth but also the out of steady state growth is generated by the successive emergence of new products/industries. The growth of older industries keeps declining while newer products/industries enjoy high growth. How high depends on µ and d. From the perspective of this model, it is easy to understand that historians have identified the 'leading' or 'key' industries in the process of economic growth. The best known example would be perhaps Rostow [1960, pp.261-62] who argues that

> "The most cursory examination of the growth patterns of different economies, viewed against a background of general historical information, reveals two simple facts:
> 1. Growth-rates in the various sectors of the economy differ widely over any given period of time;
> 2. In some meaningful sense, over-all growth appears to be based, at certain periods, on the direct and indirect consequence of extremely rapid growth in certain particular key sectors."

Vigor of the leading sectors depends on µ and d in the model. For the sake of illustration, we show a simulation result (Table 1 and Figure 4). In this example, we assume that ?, µ, and d are 0.03, 0.12 and 0.02, respectively. Table 1 and Figure 4 show both the growth rate of GDP and the average growth rate defined as $\sum_{t=1}^{t} g_t \,/\, t$ for each period (year). For the first ten years, the growth rate of the economy is higher than 9%. In the year 20, it is still 5.7%. It is the year 40 when the growth rate slows down to 3.2% which is close to the assumed asymptotic rate 3%. The average growth rate, of course, decelerates much more slowly than the growth rate itself. The average growth rate for the first thirty years, for example, is 7.5% although the growth rate in the year 30 is 3.9%. This example demonstrates that depending on µ and d, the economy can sustain a much higher growth rate than the equilibrium rate for a very long period. To repeat, the deceleration of growth comes not from diminishing returns to capital but from saturation of demand.

Everyone knows that no economy grows at 10% indefinitely. Some economies, however, actually experienced the 10% growth for a decade, and this decade long high growth is often crucial for their growth experiences. Japan, for example, kept the 10% growth for a decade and a half from 1955 through 1970. As of 1955, almost a half of working population in Japan was in agriculture. The era of high economic growth had transformed a semi-traditional economy into a modern industrial nation. We cannot dismiss 'out of steady state' merely as transitory, but must attach equal importance to it as to the steady state.

The out of steady state growth path illustrated in Figure 4 is qualitatively similar to that obtained in the old Solow [1956] model; Namely the growth rate decelerates over time. The mechanism is different, however. In the Solow model, diminishing returns to capital in production is the factor to bring about slower growth. To be specific, Mankiw, Romer and Weil [1992] show that in the standard Solow model, the growth rate *g* satisfies the following equation:

$$g_t = \frac{\dot{Y}_t}{Y_t} = n(1-a)[\log Y^* - \log Y_t]$$

Here Y^* is the steady state level of per-capita *Y*, *n*, the growth rate of population, and a is the capital elasticity. Starting with the initial value Y_0 below Y^*, the growth rate decelerates over time. The extent of deceleration depends on the capital elasticity a.

In contrast, in the present model the deceleration of the out of steady state growth rate comes from saturation of demand. To be specific, as equation (17) shows, the out of steady state growth path depends on µ and d which determine how soon demand reaches its saturation.

B. An Extension: The Non-Poisson 'Polya urn' model

In the model above, we assumed that the instantaneous probability that a new good (or sector) emerged in the process of production of each existing good obeyed the Poisson distribution with the parameter ?. This is the standard assumption in the literature. However, it is interesting to explore what happens for the ultimate growth when the probability gets smaller and smaller as time goes on. This question is important when opportunities for innovations diminish as time goes by. It is in fact often suggested that growth of 'mature' or 'old' economy slows down because such opportunities diminish. Kuznets [1953], for example, argues that

> "In the industrialized countries of the world, the cumulative effect of technical progress in a number of important industries has brought about a situation where further progress of similar scope cannot be reasonably expected. The industries that have matured technologically account for a progressively increasing ratio of the total production of the economy. Their maturity does imply that economic effects of further improvements will necessarily be more limited than in the past."

Based on the American experiences, McLaughlin and Watkins [1939] share this kind of pessimism.

Since the Poisson distribution is so commonly assumed for a success in R & D in the existing literature (e.g. Grossman and Helpman [1991] and Aghion and Howitt [1992]), it is interesting to check what happens for economic growth when the assumption does not hold. To answer this question, we analyze a discrete-time model.

In place of the Poisson distribution, we assume that the probability that a new good or sector emerges at t, p_t is

$$p_t = \frac{w}{w+t} \quad (? > 0,\ t=1, 2, \ldots)$$

This probability decreases in t, and declines asymptotically to zero. This kind of model often called 'Polya-like urns' is extensively used in population genetics; See, for example, Hoppe [1984].

To simplify our presentation, we assume first that a new good is invented 'exogenously' with p_t rather than as a branch off from the existing goods, namely that p_t is independent of the number of existing goods. In this case, when we denote the probability that there are N goods at t by $Q(N,t)$ as we did it previously, then $Q(N,t)$ satisfies

$$Q(N,t+1) = (1-p_t)Q(N,t) + p_t Q(N-1,t)$$

$$= \left(\frac{t}{w+t}\right)Q(N,t) + \left(\frac{w}{w+t}\right)Q(N-1,t)$$

for t=1, 2, …

with the following boundary conditions:

$$Q(1,t) = \left(\frac{1}{w+1}\right)\left(\frac{2}{w+2}\right)\cdots\left(\frac{t-1}{w+t-1}\right)$$

and

$$Q(t,t) = \frac{w^t}{w(w+1)(w+2)\cdots(w+t-1)} = \frac{w^t}{[w]^t}$$

where $[w]^t$ is defined by the equation.

The solution of this equation is

$$Q(k,t) = \frac{c(t,k)w^k}{[w]^t}$$

where $c(t, k)$ is the absolute value of the Sterling number of the first kind: See Aoki [1997, p. 279] or Abramovitz/Stegun [1968, P. 825]. Using the generating function

$$[x]^k = \sum_{j=0}^{k} c(k,j)x^j$$

we obtain the expected value of GDP, $Y(t)$ as

$$Y(t)=\sum_{\ell=1}^{t}\sum_{j=1}^{\ell}\frac{c(\ell-1,j-1)w^{j-1}}{[w]^{\ell-1}}\left(\frac{w}{w+\ell}\right)y(t-\ell)$$
$$=\sum_{\ell=1}^{t}\left(\frac{w}{w+\ell}\right)y(t-\ell)$$

Here $y(t-\ell)$ is the production of the final good which emerged at time ℓ. Note that $y(t-\ell)$ follows the logistic curve, and therefore, that its growth rate eventually declines to zero.

For simplicity, take ? as an integer. Then we have

$$\sum_{\ell=1}^{t}\left(\frac{w}{w+\ell}\right)=\sum_{m=1}^{w+t}\frac{1}{m}-\sum_{m=1}^{w}\frac{1}{m}\cong\log\left[\frac{w+t}{w}\right]$$

Therefore we have shown that in the present case, GDP grows drawing the logarithmic curve.

$Y(t)\sim\ \log(t+?)$

The growth rate of the economy is $1/\{(t+w)\log(t+w)\}$, and goes asymptotically down to zero.

Under the present assumption that the probability that a new good/sector emerges declines and goes asymptotically to zero, it is natural that the expected growth rate of the economy also goes asymptotically down to zero. In the standard Poisson case where the probability remains constant, the positive growth rate is sustained. We have demonstrated that it is not sustainable when the opportunities for innovations diminish. Solow [1994] makes a similar point with respect to endogenous innovations:

> "Suppose that the production function is $AF(K, L)$ where A carried (Hicks-neutral) technological progress. Successful innovations make A larger. But how much larger? .... If an innovation generates a proportionate increase in A, then we have a theory of easy endogenous growth. Spend more resources on R & D, there will be more innovations per year, and the growth rate of A will be higher. But suppose that an innovation generates only an absolute increase in A: then greater allocation of resources to R & D buys a one-time jump in productivity, but not a faster rate of productivity growth."

The standard Poisson assumption corresponds to the case of a proportionate increase in A whereas the present analysis corresponds to the case of an absolute increase in A. In the latter, the growth rate eventually declines to zero.

This analysis provides a new perspective to the issue of convergence among countries. It is usually analyzed in terms of whether the capital elasticity of production function is one or less than one. However, the issue also rests on whether opportunities for innovations diminish or not.

III. Two Macroeconomic Models: Foundations for the Logistic Growth of Demand

Having found the growth rate of GDP, we next turn to the general equilibrium of the model explained in Section I. Specifically, we must consider the consumer behavior which leads to the logistic growth of demand for an individual final good. This consumer behavior must be consistent with growth of income (equation (19)), and also with the optimum capital accumulation (equation (12)). We must recall that final goods are not only consumed but also used for capital accumulation. We suggest two different models, one the standard Ramsey model with the representative consumer and the other with diffusion of final goods among different households.

A. The Ramsey Model

As explained in II. A, the growth rate of $Y(t)$, g_t satisfies equation (18) or (19). Starting with high growth rate, it eventually decelerates and converges to ?, the probability rate that a new good/sector emerges. Therefore, the expected production of the intermediate good X also grows at g_t. Given production function (8), capital stock K also grows at g_t. Then for this time path of capital to be consistent with the firm's optimization, the interest rate r_t must satisfy (12). If the interest rate is lower than the equilibrium rate which satisfies (12), excess investment would be made, and excess supply of the intermediate good would ensue. Correctly anticipated, the value of the firm would soon decline to make r_t higher until r_t satisfies (12), and *vice versa*.

In the neoclassical approach, the queen of the economy is consumer. Demand for final goods must be, therefore, consistent with consumer's utility maximization. In what follows, we demonstrate that demand for final goods which obeys the logistic equation is in fact consistent with the intertemporal utility maximization of the Ramsey consumer with a particular utility function.

For convenience, we consider the representative consumer's utility maximization at time 0. At time 0, there is only one final good as is assumed in section II. This assumption is made just for simplicity. The assumption that there are *n* goods (*n* > 0) at time 0 merely deprives our presentation of its simplicity without giving us any additional insight.

Starting with one final good at time 0, new goods keep emerging. The probability that there are *N* goods at time *t* and the *N* + 1-th good emerges during *t* and $t + \Delta t$ is given by (15). Thus as of time 0, the consumer faces uncertainty concerning the timing of the emergence of new goods. We assume that the consumer maximizes the expected utility

$$(20) \qquad U = \int_0^\infty \left\{ \int_0^t \sum_{N=1}^\infty [\, lNe^{-lt}(1-e^{-lt})^{N-1} u_t^t (C_{N+1}(t))] dt + u_0^t (C_1(t)) \right\} e^{-qt} dt$$

where ? is the subjective discount rate and $C_j(t)$ is the consumption of the *j*-th good. In (20), the expected value is taken with respect to the probability of the emergence of new goods. A similar assumption is made in Aghion and Howitt [1992].

To obtain the logistic demand function, we assume that the utility coming from consumption of a certain final good at time *t* depends not only on *t* but also on t (t< *t*), the time when this final good emerged. To be specific, we assume that the utility function $u_t^t(C_N(t))$ is common for all the $C_N(t) \quad (N = 1,2,\cdots)$ and is

$$(21) \qquad u_t^t (C_N(t)) = h(t) \left[\frac{m}{(\, d + (m - d)e^{-m(t-t)})} \right] \log(C_N(t))$$

The logistic growth of demand (16) characterized by two parameters µ and d translates itself into utility function (21). It is actually more accurate to say that µ and d which characterize the time-dependent utility function, under suitable assumptions about $h(t)$, lead us to the logistic growth of demand. The logistic part of utility function (21), implies that the (marginal) utility coming from consumption of a particular final good depends crucially on how long time has passed since this final good first emerged. Though it monotonically increases over time, its growth rate eventually decelerates and is bound to approach zero. The elasticity of intertemporal substitution for utility function (21), s is

$$\frac{1}{s} = 1 - \left[\left\{ \frac{\dot{h}(t)}{h(t)} + \frac{m(m-d)e^{-m(t-t)}}{(\, d + (m-d)e^{-m(t-t)})} \right\} \Big/ (\dot{C}_N(t) / C_N(t)) \right]$$

It is simply one for time-independent logarithmic utility function, but depends on both *t* and t for utility function (21). We will shortly discuss ?(*t*) in (21) which is necessary to make sure the goods market equilibrium; final goods are used not only for consumption but also for investment in the intermediate good industry. So much for the expected utility which the representative consumer maximizes.

The consumer owns the stock (capital) of the intermediate good industry, S_t which earns the rate of return r_t. Thus his/her budget constraint is

$$(22) \qquad \dot{S}_t = r_t S_t - \sum_{i=1}^\infty C_i(t)$$

The consumer maximizes (20) subject to (22) and S_0. Introducing the costate variable (shadow price of capital stock) $n(t)e^{-qt}$, we obtain the necessary conditions for optimality as follows:

$$(23) \qquad C_1(t) = \left(\frac{h(t)}{n(t)} \right) \left[\frac{m}{(\, d + (m-d)e^{-mt}} \right]$$

$$(24) \qquad C_{N+1}(t) = \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} \left(\frac{h(t)}{n(t)} \right) \left[\frac{m}{(\, d + (m-d)e^{-m(t-t)}} \right] dt \quad \text{for N? 1}$$

(25) $$\frac{\dot{n}(t)}{n(t)} = q - r_t$$

and

(26) $$\lim_{t\to\infty} n(t)e^{-qt}S(t) = 0$$

Since $S(t)$ grows asymptotically at the rate of $g(t)$ and $n(t)$ satisfies (25), the transversality condition (26) is equivalent to

(27) $$\lim_{t\to\infty} \exp\left\{-\int_0^t (r_t - g_t)dt\right\} = 0$$

where g_t satisfies (19). Condition (27) is satisfied when the optimal solution (12) exists for investment decisions.

From (23) and (24), we obtain

(28) $$C(t) = \sum_{j=1}^{\infty} C_j(t) = \left(\frac{h(t)}{n(t)}\right)\left\{\frac{m}{(d+(m-d)e^{-mt})} + \sum_{N=1}^{\infty}\int_0^{\infty} lNe^{-lt}(1-e^{-lt})^{N-1}\left[\frac{m}{(d+(m-d)e^{-m(t-t)})}\right]dt\right\}$$

for total consumption $C(t)$ at time t. Thanks to (17), we further rewrite (28) as

(29) $$C(t) = \left(\frac{h(t)}{n(t)}\right)Y(t)$$

$h(t)/n(t)$ is the average propensity to consume. For clearness, we introduce a new variable $u(t)$ which is equal to the shift parameter of utility function? adjusted by $Y(t)$:

(30) $$u(t) = h(t)Y(t)$$

Then, from (25), (29), (30), the optimal aggregate consumption must satisfy:

(31) $$q - \frac{\dot{u}(t)}{u(t)} + \frac{\dot{C}(t)}{C(t)} = r_t$$

Equation (31) is nothing but the standard Euler equation or the Keynes/Ramsey rule. It requires that for optimality, the marginal rate of substition defined by the left hand side of equation (31) must be equal to the interest rate ?. Equations (31) and (12), namely optimality conditions for consumption/saving on the one hand, and for investment on the other simultaneously determine the equilibrium time paths for the interest rate r_t and the growth rate of the economy g_t.

The simultaneous determination of these two variables can be most easily seen for the steady state. In the steady state, the optimality condition for investment (12) is reduced to

$$j'(g) = \frac{P_X - j(g)}{r - g}$$

or

(32) $$\frac{P_X - j(g) + j'(g)\cdot g}{j'(g)} = r$$

The numerator of (32), $P_X - j(g) + j'(g)g$ is the marginal return of capital stock K including savings of the adjustment cost of future investment. The inverse of $j'(g)$, on the other hand, is the amount of final goods required for increasing $\dot{K}$ marginally. The left hand side of (32) is, therefore, the marginal return on investment. It depends on the growth rate g. For the optimum, it must be equal to the interest rate ?. This relation between g and ? is drawn as a downwardly sloped curve in Figure 5. As g approaches 0, ? approaches P_X. For the optimum to exist, the interest rate ? must be higher than the growth rate g. On the boundary where g is equal to ?, g = ? is $j^{-1}(P_X)$.

In the steady state, the growth rate of consumption must be equal to g. Therefore the optimality condition for consumption/saving, namely the Euler equation (31) is reduced to

$$q - u^* + g = r$$

where u^* is the steady rate of shift in utility function $u^* = \dot{u}(t) / u(t)$.

Figure 5 plainly shows that for the equilibrium to exist, thefollowing inequality must hold :

$$0 < q - u^* < P_X$$

This inequality implies that u^* must be smaller than the consumer's discount rate ?. Figure 5 also shows that when the equilibrium exists, it is unique.

Now it should be obvious that for the equilibrium growth explained above to be consistent with the model presented in sections I and II, a time-dependent shift parameter of utility function $u(t)$ cannot be arbitrary. Specifically, the growth rate g_t given by equation (19) must be equal to the growth rate determined by equations (12) and (31). From (12) and (31), we can draw a conclusion that for this proposition to be true, $u(t)$ must satisfy

$$\frac{\dot{u}(t)}{u(t)} = -\left\{\left[\frac{j\,'(g(t))}{(1 - j\,(g(t))}\right] + \left[\frac{j\,''(g(t))}{j\,'(g(t))}\right]\right\}\dot{g}(t) - \left[\frac{P_X - j\,(g(t))}{j\,'(g(t))}\right] + q$$

Recall the definition of $u(t)$, (30), and we know that this equation is equivalent to

$$\frac{\dot{h}(t)}{h(t)} = -\left\{\left[\frac{j\,'(g(t))}{(1 - j\,(g(t))}\right] + \left[\frac{j\,''(g(t))}{j\,'(g(t))}\right]\right\}\dot{g}(t) - \left[\frac{P_X - j\,(g(t))}{j\,'(g(t))}\right] - g(t) + q$$

g_t in this equation satisfies (19), and the shift parameter of utility function (21) $h(t)$ is a well-defined function of time. In the steady state where $\dot{g}(t) = 0$ and g =?, the following equation must hold for h^*, the steady rate of shift in utility function (21), $h^* = \dot{h}(t) / h(t)$:

$$h^* = q - \left[\frac{P_X - j\,(\,l)}{j\,'(\,l)}\right] - \,l \tag{33}$$

It is not surprising that $h(t)$ must be a specific function of time because we assumethat the growth of each final good is a particular function of time, namely the logistic equation with specific parameters µ andd, and the convex adjustment cost of investment has a particular functional form, $j\,(z)$, and that a birth of new good/industry also obeys a particular stochastic process. Recall that the logistic growth has been actually observed for so many goods and industries, and that to take it as a stylized fact was our starting point.

Obviously, a particular form of $h(t)$ is of little interest. We merely showed the existence of utility function with which the optimization of the consumer leads to the logistic growth of demand for each final good. There is, however, an important generic property of the utility function analyzed here; That is, h^* depends on ?, $¶h^* / ¶\,l > 0$. Out of steady state, $h(t)$ depends not only ? but also on µ and d. It is only natural that a combination ofµ and d which generates higher growth corresponds to greater $\dot{h}(t) / h(t)$. In the steady state, however, constant $h^* = \dot{h}(t) / h(t)$ becomes independent of µ and d, and an increasing function of only ? (equation (33)). In section II, we have established that the growth rate of the economy is asymptotically equal to?, the probability that a new good or industry emerges in the existing process of production. Within the Ramsey framework, we interpret this result as indicating that emergences of new goods/industries make the growth rate of the economy higher by way of raising otherwise declining marginal utility of consumption. In this sense that we can call ? *demand creating innovations*.

The saving rate in this model is equal to $j\,(g)$, and therefore, is an increasing function of the growth rate. The positive relation between growth and saving is empirically well documented. Bosworth [1993], for example, in a comprehensive study of the determinants of saving rates in OECD countries from the 1960's to the 1980's, finds that the growth rate of income is the most important determinant of saving. When growth is due mainly to capital accumulation, such a positive correlation is not surprising. However, when technicalprogress is the engine of growth, the matter is theoretically not so clear.

For example, in the standard Ramsey model with exogenous technical progress, the effective discount rate becomes $q + (s - 1)\,g$ where ? is the subjective discount rate, s the elasticity of intertemporal substitution, and ? is the rate of labour-augmenting technical progress. It is, therefore, quite possible that saving declines when the productivity growth rises[2]. Such a result is not so surprising in standard models because growth is nothing but piling up the essentially same commodity; technical progress would discourage patience in such a world. In contrast, ? in the present model creates new goods the marginal utility of which is higher than that of the existing 'old' goods. Thus, utility function shifts depending on ?. High ? keeps the saving rate high, and thereby the growth rate.

Let us sum up the Ramsey model. Investment in the intermediate good sector is made so as to maximize the stock price of the firm whereas consumption/saving is determined by the consumer's intertemporal utility maximization. The allocation between consumption and investment is adjusted by the interest rate. The generating force of the economy is ever changing consumer's preference. Since the marginal product of capital does not decline, the economy grows whenever capital accumulates. Capital accumulates when demand grows. Growth of demand, in turn, depends crucially on kinds of available goods and their ages. Under the assumed structure of preferences, demand for each good initially grows fast but its growth eventually slows down to zero. Innovations, namely emergences of new goods/industries sustain growth by sustaining otherwise declining marginal utility of consumption. The steady state growth rate is, in fact, equal to the power of demand creating innovations ?.

B. Diffusion of Final Goods among Different Households

The Ramsey model is the most standard approach in macroeconomics. However, in many economies, for many periods in history, a declining growth of demand for a particular product has been very closely related to diffusion of the product among different households. Some households own the product while others do not. It is particularly true for such consumer durables as television, refrigerator, car and personal computer. For these consumer durables, it makes more sense to analyze their growth in a model with different households than in a model with the representative consumer. In this section, we consider such a model.

Suppose that there are M households in the economy. Without loss of generality, we can assume that M is equal to µ/d. Households are indexed by i (i=1, 2, …, M=µ/d). We define $f_{iN}(t)$ functions:

$f_{iN}(t) = 1$ if household i purchases the N-th product at time t.

$f_{iN}(t) = 0$ if household i does not purchase the N-th product at time t.

We assume that the number of households which consume the N-th product at time t, $m_N(t)$ follows a birth and death process with the birth rate µ and the death rate dm_N. Note the following relation:

$$\sum_{i=1}^{M} f_{iN}(t) = m_N(t) \tag{34}$$

As explained in section I, this process leads us to the logistic equation for the expected value of $m_N(t)$, $\hat{m}_N(t)$. Thus if the N-th product emerged at t, $\hat{m}_N(t)$ satisfies the following equation:

$$\hat{m}_N(t) = \frac{m}{(d + (m - d)e^{-m(t-t)})} \quad \text{for each } N. \tag{35}$$

The (expected) diffusion rate or the percentage of households which consume the N-th product is $\hat{m}_N(t) / M$.

For simplicity we assume that household purchases $1 - s$ unit of any final product if it consumes this product. s is the saving rate. As in Sollow [1956], the present analysis abstracts from the determination of s. The saving rate is assumed to be common for all the households (i=1, 2, …, M), and depends positively on the interest rate r_t. It also depends on time t. Note that the consumption of a final good by each household is constant. This assumption seems to hold, as an approximation, for many consumer durables.

Then, with the expected income of household i, $I_i(t)$, the budget constraint for household i at time t becomes

[2] Indeed, a simulation analysis of Carroll and Weil [1993] demonstrates that the medium-run relationship between growth and saving is negative under the assumption of s =1 and s =4, and that if s =4 the long-run relationship is negative as well.

$$
(36) \quad \begin{aligned} I_i(t) &= \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1}(1-s)\, f_{iN+1}(t)dt + (1-s)\, f_{i1}(t) + sI_i(t) \\ &= \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} f_{iN+1}(t)dt + f_{i1}(t) \end{aligned}
$$

Thanks to (34) and (35), incomes of all the households (36) sum up to GDP, $Y(t)$:

$$
\begin{aligned} \sum_{i=1}^{M} I_i(t) &= \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} \sum_{i=1}^{M} f_{iN+1}(t)dt + \sum_{i=1}^{M} f_{i1}(t) \\ &= \sum_{N=1}^{\infty} \int_0^t lNe^{-lt}(1-e^{-lt})^{N-1} \frac{m}{(d+(m-d)e^{-m(t-t)})} dt + \frac{m}{(d+(m-d)e^{-mt})} \\ &= Y(t) \end{aligned}
$$

The goods market equilibrium is then

(37) $\quad s(r_t, y(t)) = j(g_t)$

$y(t)$ is a time dependent shift parameter of the saving rate $(\P s / \P y > 0)$, and is equivalent to $h(t)$ in the Ramsey model.

The interest rate r_t and the growth rate g_t are simultaneously determined by (12) and (37) in such a way that g_t satisfies equation (19) in section II. In the steady state,

$$
s(r^*, y^*) = j(l)
$$

and (32) hold. The two equations imply that the shift parameter of s, y^* is an increasing function of ?. The more new goods emerge, the higher is the saving rate. Who would keep saving if there were only rice on earth?

This mechanism is basically the same as that in the Ramsey model. However, unlike the Ramsey model, the generating force of growth in the present model is diffusion of goods among different households rather than the shift of preference of the representative consumer. Diffusion of goods among households creates demand, which in turn induces capital accumulation and growth. Growth, on the other hand, creates higher income which makes more households afford to purchase goods. Equation (36) defines income distribution which generates diffusion of final goods among households. Because the amount of a final good which each household purchases is bounded, growth of production of an individual good necessarily decelerates parallel to diffusion of the good among households. Creation of new goods is the ultimate factor to sustain growth in such an economy.

IV. Concluding Remark

In the standard literature, the fundamental factor to restrain economic growth is diminishing returns to capital. We presented a model in which the factor to restrain growth was saturation of demand. Our analysis began with a common observation that for individual products/industries, there was a history of logistic development with initial acceleration and eventual retardation of growth. Taking it as a 'stylized fact', in this paper, we presented a formal model of growth consistent with this important 'fact'.

The existing literature on growth (e.g. Grosman and Helpman [1991], and Aghion and Howitt [1992]) gives firm microeconomic foundations for R & D activities, and by so doing leads us to a conclusion that the endowment of production factors used in R & D is conducive to innovation and growth. The relation between innovation and growth has been already much studied from this perspective. In stead of microeconomic foundations for R & D, our analysis focused on the effects of the emergence of new products/industries on the growth of demand.

In terms of the standard Ramsey model, it highlights the time-dependence of the utility function; the utility function changes over time depending on the kind and the age of a product. The assumption is consistent with a S-shaped life cycle of an individual product/industry which is nothing but our starting point. We also presented a model in which a S-shaped growth of an individual product/industry reflected a diffusion of the product among different households rather than a shift of preference of the representative consumer. The rich in less developed countries often attains the standard of living comparable to that of well developed countries. Economic growth raises the average standard of living by way of diffusion of new products among different households. This common observation leads us to the second model with heterogeneous households.

Under the assumption of a unitary elasticity of capital in production function (dubbed the AK model), the economy grows whenever capital accumulates. However, the growth of demand for an individual product/industry is, bound to slow down and fall ultimately to zero. And it restrains capital accumulation. Growth of demand revives when major new product or industry emerges. Technical progress creates demand. Then capital accumulates, and the economy grows. The ultimate factor to sustain growth in steady state is the rate of emergence of new products/industries or ? in the model.

This model provides new perspectives to several important problems addressed by the economics of growth. First and most important is the nature of technical progress or innovations. In the standard analysis, technical progress brings about higher value added given the same level of inputs. It is basically equivalent to an ' upward shift' of production function. The so called product variety model (Grossman and Helpman [1991]) using the Dixit/Stiglitz production/utility function, successfully endogenizes this kind of technical progress. Empirically technical progress has been measured by growth accounting as TFP (total factor productivity).

In the model presented here, the aggregate production function is $Y = AK$. Since A is constant, there is no TFP growth; For the economy to grow, capital must accumulate. Innovation or technical progress in this model creates major new product or industry which commands high growth of demand, thereby induces capital accumulation, and revives economic growth. Schumpeter [1934] in his famous book, distinguishes five types of innovations: (1) the introduction of a new good, (2) the introduction of a new production method, (3) the opening of a new market, (4) the conquest of a new source of supply of raw materials, and (5) the new organization of industry. His first and third types of innovations as an engine for growth seem to be most naturally interpreted in terms of a kind of model presented here.

The distinction between the conventional TFP and demand creating technical progress is not only theoretically important but is also empirically relevant. Young [1995], for example, in his careful study of growth accounting, demonstrates that TFP growth in the newly industrializing countries (NICs) of East Asia, i.e. Hong Kong, Singapore, South Korea, and Taiwan, is not so extraordinarily high but actually comparable to those in other countries. The very high average growth rate (8 to 10%) of East Asian NICs for such a long period as 25 years must be, therefore, explained by extraordinary injection of capital and labor rather than extraordinary TFP growth. Still the basic question remains; Why did the economy grow so fast in these countries? The analysis of this paper suggests that in these countries, new sectors which command high growth of demand vigourously emerged (High μ/d and ?). TFP growth may not be extraordinarily high, but it does not necessarily mean the absence of demand creating innovations. Young, in fact, reports that in East Asian NICs the industrial structure has drastically changed. A change in the industrial structure in the course of economic growth is likely to reflect demand creating innovations which are conceptually different from TFP. To the extent that exports happen to have commanded high growth (i.e. high μ/d), we can easily understand that high growth of East Asian NICs often looked like export-led.

The present paper also provides a new perspective to the issue of convergence among countries. In the standard literature, the basic factor to force convergence is diminishing returns to capital. The out of steady state growth path of the present model, illustrated in Figure 4, is qualitatively similar to that obtained in the standard Solow model. And yet the mechanism is different. In the present model, the basic factor to bring about convergence is saturation of demand rather than diminishing returns to capital in production.

The endogenous growth literature demonstrated that innovations could be the ultimate factor to sustain growth. In the present model, too, under the standard Poisson assumption of constant ?, the positive growth rate is sustained. However, we demonstrated in section II. B. that when opportunities for innovations diminished, the growth rate went asymptotically down to zero. In this case, despite the AK production function, the convergence among countries would occur. Limits to further technical progress, which are assumed away by the Poisson assumption in the standard literature, deserve serious investigation.

The model has an obvious policy implication. For various reasons, subsidies for R and D can be justified. Then from the perspective of this model, it is important to subsidize R and D which would bear a new product or industry for which high growth of demand is expected. The targeted industrial policy of Japan's Ministry of International Trade and Industry (MITI) had high income elasticities of demand as a most important criterion for subsidy.

Finally, in the present paper to make our analysis tractable, we inevitably made an unrealistic assumption that μ, d, and ? were constant. We hope that the assumption is justified for the purpose of studying economic growth. However, in the short/medium run μ, d and ? would all fluctuate. Giving the μ, d and ? shocks to the model economy, as is done in the standard real business cycle (RBC) literature, one would be able to generate fluctuations of the growth rate. Such simulation exercises with 'demand shocks' might generate a more realistic explanation of short-run fluctuations than that based on TFP shocks.

The model presented in this paper is based on the idea that retardation of growth of demand sets limits to growth rather than diminishing returns on capital in production. In addition to the conventional TFP, technical progress also creates demand, thereby induces capital accumulation, and ultimately sustains economic growth. As we explained it in this section, this model provides new perspectives to the analysis of economic growth.

Appendix

Equation (13) in the text can be solved in the following way. First we define the generating function $G(z, t)$ as

$$G(z,t) = \sum_{n=N_0}^{\infty} z^n Q(n,t)$$

Multiplying (13) by z^n, and taking its sum over $n = N_0, N_0 + 1, \cdots,$ we obtain the partial differential equation:

$$\text{(A.1)} \qquad \frac{\partial G(z,t)}{\partial t} = lz(z-1)\frac{\partial(G,t)}{\partial z}$$

with the initial condition

$$\text{(A.2)} \qquad G(z,0) = z^{N_0}$$

To solve this partial differential equation, we introduce the artificial variable called s for which the following ordinary differential equations hold.

$$\text{(A.3)} \qquad \frac{dt}{ds} = 1$$

$$\text{(A.4)} \qquad \frac{dz}{ds} = -lz(z-1).$$

With the initial condition $(s, t) = (0, 0)$, (A.3) can be solved immediately to give

$$t=s$$

Similarly, with the initial condition $(s, z) = (0, m)$, (A.4) can be solved to give

$$\text{(A.5)} \qquad ls = \log\left(\frac{z}{z-1}\right)\left(\frac{m-1}{m}\right)$$

Since $s = t$, from (A.5) we obtain

$$\text{(A.6)} \qquad m = \frac{e^{-lt}z}{\left[1-\left(1-e^{-lt}\right)z\right]}$$

On the other hand, from (A.1), (A.3) and (A.4) we know that $G(z(s), t(s))$ satisfies

$$\begin{aligned}\frac{dG}{ds} &= \frac{\partial G}{\partial z}\cdot\frac{dz}{ds} + \frac{\partial G}{\partial t}\cdot\frac{dt}{ds} \\ &= -lz(z-1)\frac{\partial G}{\partial z} + \frac{\partial G}{\partial t} = 0\end{aligned}$$

and therefore that G as a function of s is constant. Since z is m when s is zero, from (A.2) we find that this constant is m^{N_0}. Using (A.6), we see that $G(z, t)$ is

$$\text{(A.7)} \qquad G(z,t) = \frac{e^{-lN_0 t}z^{N_0}}{\left[1-\left(1-e^{-lt}\right)z\right]^{N_0}}$$

The denominator of (A.7) can be expanded as

$$\begin{aligned}\frac{1}{\left[1-\left(1-e^{-lt}\right)z\right]^{N_0}} &= \sum_{\ell=0}^{\infty}\binom{-N_0}{\ell}(-1)^{\ell}\left(1-e^{-lt}\right)^{\ell} z^{\ell} \\ &= \sum_{\ell=0}^{\infty}\left(\frac{N_0+\ell-1}{\ell}\right)\left(1-e^{-lt}\right)^{\ell} z^{\ell}\end{aligned}$$

Thus

$$\text{(A.8)} \qquad G(z,t) = \sum_{\ell=N_0}^{\infty}\binom{\ell-1}{\ell-N_0} e^{-lN_0 t}\left(1-e^{-lt}\right)^{\ell-N_0} z^{\ell}$$

The probability that the number of final goods at t is N, $Q(N, t)$ is the coefficient of z^N of this generating function (A.8), and is given by

$$Q(N,t) = \binom{N-1}{N-N_0} e^{-lN_0 t}\left(1-e^{-lt}\right)^{N-N_0}$$

For simplicity, if we take N_0 as 1, $Q(N, t)$ becomes

$$Q(N,t) = e^{-lt}(1-e^{-lt})^{N-1}$$

This is equation (14) in the main text, and is called the negativebinominal distribution.

References

Abramovitz, M., and I. A. Stegun, *Handbook of Mathematical Functions*, New York: Dover Publication Inc., 1968.

Aghion, P. and P. Howitt, "A Model of Growth through Creative Destruction", *Econometrica*, March, 1992.

Aoki, M., *New Approaches to Macroeconomic Modeling*, Cambridge: Cambridge University Press, 1996.

------,"Shares in Emergent Markets: Dynamics and Statistical Properties of Equilibrium Classification of Agents in Evolutionary Models" in T. Katayama and S. Sugimoto (eds.), *Statistical Methods in Control and Signal Processing*, New York: Marcel Dekker, 1997.

Arrow, K., "The Economic Implications of Learning by Doing," *Review of Economic Studies*, June, 1962.

------, "A Classificatory note on the production and transmission of technological knowledge", *American Economic Review*, May, 1969.

Bosworth, B., *Saving and Investment in a Global Economy*, Washington D. C.: Brookings Institution, 1993.

Carroll, C. and D. Weil, "Saving and Growth," Cambridge, MA: NBER Working Paper no. 4470, 1993.

Grossman, G. M. and E. Helpman, *Innovation and Growth in the Global Economy*, Cambridge, Mass.: MIT Press, 1991.

Hoppe, F. M., "Polya-like urns and the Ewens' sampling formula", *Journal of Mathematical Biology*, 20, p. 91-94, 1984.

Kuznets, S., *Economic Change*, New York: Norton, 1953.

Maddison, A., *Phases of Capitalist Development*, New York: Oxford University Press, 1982.

Mankiw, N. G., D. Romer, and D. Weil, "A Contribution to the Empiric of Economic Growth," *Quarterly Journal of Economics*, May, 1992.

McLaughlin, G. and R. Watkins, "The Problem of Industrial Growth in a Mature Economy," *American Economic Review*, March, 1939.

Montroll, E. W., "Social Dynamics and the Quantifying of Social Forces," *Proceedings of National Academy of Sciences U. S. A.*, Vol. 75, No. 10, October, 1978.

Oi, W. Y., "The Welfare Implications of Invention," in T. Bresnahan and R. J. Gordon eds., *The Economics of New Goods*, Chicago: University of Chicago Press, 1997.

Romer, P., "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, October, 1986.

Rostow, W. W., *The Process of Economic Growth*, Oxford: Oxford University Press, 1960.

------, *The World Economy: History and Prospect*, Austin: University of Texas Press, 1978.

Schumpeter, J., *Theory of Economic Development*, Cambridge, MA: Harvard University Press, 1934.

Solow R., "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, February, 1956.

------, "Perspectives on Growth Theory," *Journal of Economic Perspectives*, Winter, 1994.

Stokey, N., "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy*, August, 1988.

Uzawa, H., "Time Preference and the Penrose Effect in a Two-class Model of Economic Growth", *Journal of Political Economy*, July/August, Part II, 1969.

Young, A., "Increasing Returns and Economic Progress", *Economic Journal*, December, 1928.

Young, A., "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience," *Quarterly Journal of Economics*, August, 1995.